SELECTIVE EXPERIMENTS IN PHYSICS

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EXPERIMENT No. ....

THE LAWS OF VIBRATING STRINGS

OBJECT: To study the properties of a vibrating string, and to determine the frequency of a tuning fork by tuning a string to unison with it.

METHOD: A piano wire stretched over a sounding box is tuned to unison with a tuning fork of known frequency. The tension of the wire is adjusted by means of weights suspended from it and the length of the vibrating segment is adjusted by means of movable bridges. With the tension constant the wire is tuned to several forks in succession by making adjustments of the length. The relationship between frequency and length is shown by a graph of frequency versus the reciprocal of the length. A second set of observations is made in which the tension is varied, the length being kept constant. The square of the frequency is plotted against the tension. The frequencies computed from measured values of the length, tension and mass per unit length are compared with the values stamped on the forks.

THEORY: The vibration of a stretched string is a case of standing waves. When a string is set into vibration by plucking or bowing, the train of waves which is generated is reflected at the fixed ends of the string and travels to and fro with gradually diminishing amplitude. Thus there are present, simultaneously, waves travelling in opposite directions after reflection at the two ends. The vibration of the string is a composite motion resulting from the combined effect of the oppositely directed wave trains. The interaction of these oppositely directed wave trains is such that at certain equally spaced positions the displacements produced by the two waves are equal in magnitude and opposite in phase *at all times,* with the result that the two effects cancel each other and the resultant disturbance is always zero. These positions of no disturbance are called *nodes.* Midway between the nodes the phase relations are such that the resultant displacement varies periodically from zero to twice that due to one wave, and the disturbance is a maximum. These regions are called *antinodes.* The vibrating segments of string between consecutive nodes are called *loops.* The amplitude of vibration increases gradually from zero at a node to maximum at an antinode. The succession of nodes and antinodes is called a standing wave. The standing waves in a stretched string vibrating in two segments are represented diagrammatically in Fig. 1 in which the solid line represents the form of the string at an instant of maximum displacement and the dotted line represents the configuration one half-period later when the displacements are reversed. The internodal distance is represented by l. From the fact that the phase of the disturbance in the antinode A_1 is opposite to that in A_2, it is evident that the wave length λ (distance between succes-

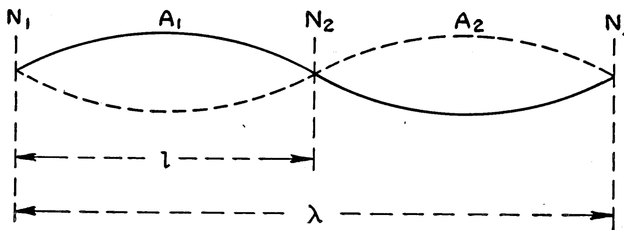

Fig. 1. Standing wave in a string.

sive particles in the same phase) includes two internodal distances. This relationship is an important one in the experimental study of standing waves in any medium.

The frequency of vibration of a string (which determines the pitch of the note emitted) depends upon the length of the string, its mass per unit length, and the tension to which it is subjected. To deduce the relationship among these quantities, begin with the fundamental equation of wave motion

$$V = n\lambda \qquad (1)$$

where V is the velocity of the wave, n the frequency and λ the wave length. The validity of this equation is apparent when it is considered that if, in one second, n waves of length λ are stretched out end to end, their combined length is $n\lambda$ and the first wave must have travelled a distance $n\lambda$ from the source.

The expression for the velocity of a transverse wave in a cord can be obtained from a consideration of Fig. 2. The drawing represents a section of a cord in which a wave is travelling from right to left with a velocity V. Since it is the relative motion of the wave with respect to the string which is significant, the discussion will be simplified (and the situation essentially unaltered) if the string is regarded as moving to the right with the same numerical velocity, the wave remaining stationary. This situation could be achieved physically by drawing the cord through a glass tube curved to fit the wave form. Consider a section of the cord AB so short that it may be regarded as the arc of a circle. Let the tension at A and B (which is directed along the cord at these points) be represented by T. The resultant of these two forces is a centrally directed force F_c on the segment AB (see the vector diagram Fig. 2b). As the cord passes from A to B its velocity changes in direction, but not in magnitude, as a result of centripetal acceleration due to the force F_c

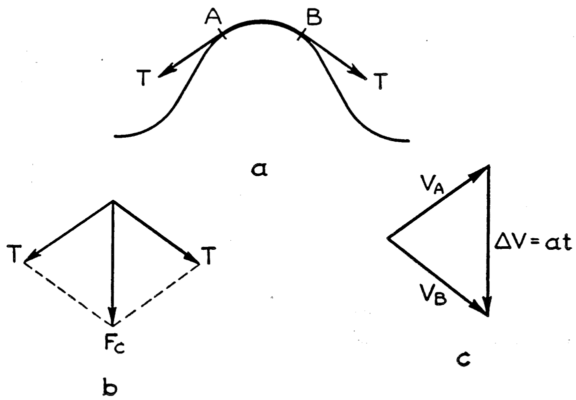

Fig. 2. Velocity of a transverse wave in a string.

(Fig. 2c). Since both tension and velocity are parallel to the cord, by similar triangles

$$\frac{T}{F_c} = \frac{V}{at} \qquad (2)$$

By Newton's second law $F_c = msa$, where m is the mass per unit length, s the length of the segment and a the centripetal acceleration. Also the distance travelled from A to B is $s = Vt$. Substituting these relationships in Eq. (2)

$$\frac{T}{maVt} = \frac{V}{at} \qquad (3)$$

from which

$$V = \sqrt{\frac{T}{m}} \qquad (4)$$

If such a cord, enclosed in a glass tube bent in an arc AB, were stationary and under tension T, there would be a normal force on the walls of the tube tending to straighten it out; but if the cord is given a velocity, centrifugal action neutralizes this force and the cord is able to maintain its configuration without the aid of the tube. Eq. (4) yields the velocity in centimeters per second when T is in dynes and m is in grams per centimeter length.

Substituting the value of V from Eq. (4) in Eq. (1) and solving for the frequency

$$n = \frac{1}{\lambda}\sqrt{\frac{T}{m}} \qquad (5)$$

Substituting $\lambda = 2l$

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad (6)$$

The maximum possible value for the internodal distance l is the length of the cord. Thus the lowest possible frequency is produced when the string vibrates in one segment. This minimum frequency is called the *fundamental* frequency of the string. Frequencies greater than the fundamental are called *overtones*. Overtones, whose frequencies are integer multiples of the fundamental, are called *harmonics*. From the symmetry of a string fixed at both ends it is apparent that all the harmonics are possible.

APPARATUS: The apparatus employed in this experiment consists of a sonometer, a set of weights, several tuning forks, a rubber hammer and a meter stick. The sonometer, two forms of which are illustrated in Figs. 3 and 4, consists essentially of one or more piano wires stretched over a sounding box. The tension in the wires is adjusted by means of weights suspended from them, and the effective length is regulated by movable bridges. In some models the tension is adjusted by means of screws, the value of the tension being indicated by spring balances attached to the wires.

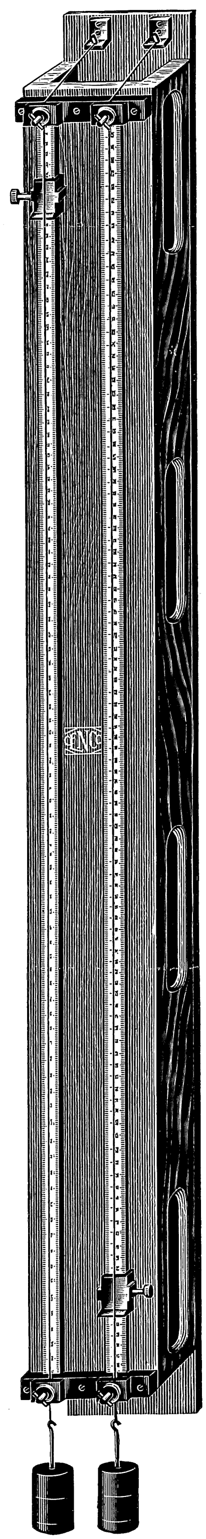

Fig. 4. Sonometer, wall model

PROCEDURE:

Experimental:

Part I. Relation between frequency and length. Locate the bridges so as to utilize most of the length of one of the wires. Sound the fork of lowest frequency by striking it with the rubber hammer, and add weights to the string until its frequency is nearly the same as that of the fork. *Caution: Never strike a tuning fork with a metal object nor allow the prongs to strike a hard body while vibrating.* By adjusting one of the bridges, tune the string exactly in unison with the fork. The inexperienced student will have to practice making this adjustment until he learns to detect a slight difference in pitch. With the ear held close to the fork and the string, pluck the latter gently and listen for beats. (Violent plucking gives rise to harmonics and spurious vibrations which make tuning difficult.) As the tuning becomes closer the frequency of beats diminishes, and when the two are in unison the beats disappear. The student whose ear is unreliable may facilitate tuning by the following method. Make a rider of a small piece of paper folded in the form of a V and place it on the wire at the midpoint. Sound the fork and place its base firmly in contact with the sounding box. Do not pluck the string. When the string is in tune with the fork, the forced vibrations set up in the sounding box by the vibrating fork will be taken up by the string and the rider will be displaced.

When the adjustment has been made, measure the length of the

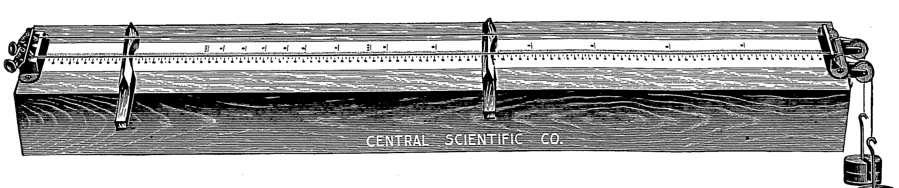

Fig. 3. Sonometer, table model

string between the bridges. Repeat the determination with the fork of next lowest frequency. Continue in this way until a total of four observations has been made. Record the data as indicated in Table I.

Part II. Relation between frequency and tension. Keeping the length constant at the last value used in Part I, reduce the tension until the string is in tune with the fork of next lowest frequency, and so on for all the forks. Record the data.

Weigh a measured length of the same kind of wire and determine the mass per unit length in grams per centimeter.

Optional Experimental:

Part III. Relation between tension and mass per unit length for a constant frequency. For this part of the experiment the sonometer must be equipped with two wires of different mass per unit length. Apply a known weight to one wire and adjust the tension of the other until it is in tune with the first one. Compare the ratio of the tensions with the ratio of the masses per unit length.

Interpretation of Data: From the data of Part I plot the frequency n as ordinate against the reciprocal length $\frac{1}{l}$ as abscissa.

From the data of Part II plot the square of the frequency n^2 as ordinate and the tension T as abscissa.

For all observations compute the frequency by Eq. (6) and compare with the value marked on the fork.

QUESTIONS: 1. Describe a simple method of using the sonometer to determine the frequency of a tuning fork by comparison with a standard fork.

2. Two wires A and B are made of the same material. A has twice the length and twice the radius of B. What is the numerical ratio of their frequencies when under the same tension?

3. Compare the tensions of a brass wire and an aluminum wire of the same length and cross section when tuned in unison.

4. A string 1 meter long, having a mass per unit length of 0.4 gm/cm, is under a tension of 5 kilograms. What is the wave length in air of the note emitted?

TABLE I

Length *cm*	*Tension* *gm—wt*	*Frequency* *sec^{-1}*	
		marked	*computed*