Vacuum Cannon Calculations

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Assumptions

These calculations make several (incorrect) assumptions:

1. There is no friction at all (including air resistance) affecting the ping-pong ball.
2. The ping-pong ball undergoes constant (uniform) acceleration.
3. The ping-pong ball's velocity is unaffected by breaking through the tape when exiting the barrel.

Numerical Values

Cannon length: $x = 121\ inches = 3.07\ meters$.
Cannon bore diameter: $d_{bore} = 1.6\ inches$.
Ping-pong ball diameter: $d = 40\,mm \rightarrow A = \pi(20\,mm)^2 = 1.256\text{x}10^3\,mm^2 = 1.256\text{x}10^{-3}\,m^2$.
Ping-pong ball mass: $m = 2.5\ grams = 2.5\text{x}10^{-3}\ kg$.
Atmospheric pressure: $P = 1\ atm = 14.7\ psi = 101,325\ N/m^2$.
Aluminum can mass: $m_{can} = 13.2\ grams$.

Calculations

Pressure = Force/Area or F = P · A:

$F = P \cdot A = (101,325\ N/m^2)(1.256\text{x}10^{-3}\ m^2) = 127.3\ N$.

Force = mass · acceleration or a = F/m:

$a = F/m = (127.3\ N)/(2.5\text{x}10^{-3}\ kg) = 5.09\text{x}10^4\ m/s^2 = 5,192\ g$.

Assuming a constant acceleration a over a distance x, $v_f^2 = v_i^2 + 2ax$, but $v_i = 0$:

$v_f^2 = 2ax$ or $v_f = \sqrt{2ax} = \sqrt{2(5.09\text{x}10^4\ m/s^2)(3.07\ m)} = 559\,m/s = 1834\ ft/s\ \approx Mach\ 1.6$.

The kinetic energy of the ping-pong ball is E = $\frac{1}{2}mv^2$:

$E = \frac{1}{2}mv^2 = \frac{1}{2}(2.5\text{x}10^{-3}\ kg)(559\ m/s)^2 = 391$ Joules.

Notes

For comparison, a .22LR rifle has a typical muzzle energy of 159 Joules and a .38 Special pistol has a typical muzzle energy of 420 Joules (source: http://en.wikipedia.org/wiki/Muzzle_energy). The ping-pong ball isn't going nearly as fast as calculated. These calculations only serve as an exercise.