CENTRAL SCIENTIFIC COMPANY

OPERATING INSTRUCTIONS

Hinged Stick And Falling Ball Demonstrator
No. 74874

1. Introduction

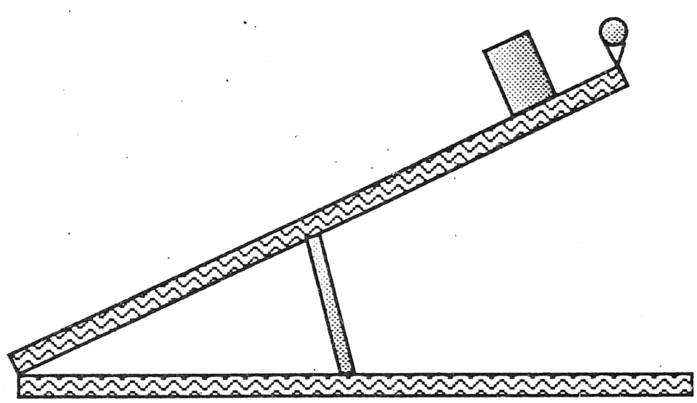
The Hinged Stick and falling Ball Demonstrator is a device which illustrates the difference between angular and linear acceleration.

2. Description

Two boards, approximately 39" long, are joined together by a hinge. A plastic cup and a golf tee are placed at one end of the upper board. A wooden prop is used to separate the boards. One rubber and one steel ball are also included.

3. Setup and Operation

The Hinged Stick is set up as shown. The foam pad in the plastic cup reduces the rebound height of the balls and is most effective when located slightly above (1/4") the bottom of the plastic cup. Colored marks show where the wooden prop is to be placed. The prop should then be removed with a quick motion of the hand. The rubber ball will then fall in the plastic cup. Try the experiment with



the steel ball. The same effect will be noticed.

4. Discussion

Consider a stick standing on end (vertically) on a table top. If it falls over, all points on it describe circular paths and all points on it have the same *angular* velocity. Clearly, however, all points on it do not have the same *linear* velocity. We assume the lower end to remain fixed, as on a hinge. (Real sticks so standing and so falling, with the lower end not hinged, experience different motions.) Because all points of the stick have the same angular velocity at all times, all points have the same

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angular acceleration at all times. If the angular acceleration is everywhere the same, then the linear acceleration is everywhere different because the linear acceleration of a particle is equal to the product of its radius from the axis of rotation and the angular acceleration ($r,\ddot{\theta}$) and every point along the stick is at a different distance from the axis. Note that the linear acceleration of all points on the stick is in a direction normal to the length of the stick.

When the hinged stick falls and the ball falls into the cup, the end of the stick must have had a greater acceleration than that of free fall. This must be so if the cup catches the ball (as it does) because the very end of the stick must have moved away from the ball!

If the length of the prop were to be varied so that the angle θ is changed, then only at angles less than some critical angle θ_0 does the stick drop away from the ball. We can deduce this result by a consideration of the equation of motion of the stick:

$$I\ddot{\theta} = -Mg\,\frac{L}{2}\cos\theta$$

This equation is merely a statement of Newton's second law of motion in the form: rate-of-change of angular momentum equals moment of applied force. We apply the minus sign since the torque is such as to reduce θ.

$$I = \frac{1}{3}ML^2$$

Where I = moment of inertia of the stick about the hinged end, L = length of stick, and M = mass of stick.

Transposing: $$\ddot{\theta} = \frac{-3g}{2L}\cos\theta$$

The linear acceleration of the end of the stick is given by:

$$\ddot{s} = L\ddot{\theta} = \frac{-3g}{2}\cos\theta$$

and the vertical component of this acceleration is given by:

$$\ddot{y} = \ddot{s}\cos\theta = \frac{-3g}{2}\cos^2\theta$$

We now ask, for what angles is this acceleration greater than the acceleration of gravity? That is:

$$\frac{-3g}{2}\cos^2\theta < -g$$

$$\cos^2\theta > \frac{2}{3}$$

This relation is satisfied by angles θ smaller than about 35°
($\theta_0 = \cos^{-1}\sqrt{2/3} \approx 35°$).

What is observed here is related also to the behavior of falling chimneys which are observed to buckle as they fall.

The phenomenon is less convincingly demonstrated as follows: On two horizontally-held fingers, F_1 and F_2, rest a meter stick with the fingers near the end of the stick. On one end of the stick place a small block *m*. Now suddenly remove finger F_1, allowing the stick to fall as if hinged at F_2. Astonishingly, the end of the stick falls away from the block!

5. Maintenance

The Hinged Stick and Falling Ball apparatus needs no special maintenance. Should a problem develop, please contact Central Scientific Company, giving details of the problem. Your business is important to us. For us to give you maximum service, do not return anything until you have received authorization from Central Scientific Company.

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